Main Steps

The 4 main steps for proving a language \( A \) is not regular is as follows:

**Step 1: Demon Picks** \( k \geq 1 \). You are given some pumping length \( k \geq 1 \).

**Step 2: You pick** \( xyz \). Select \( x, y, z \) such that \( xyz \in A \) and \( |y| \geq k \).

**Step 3: Demon Picks Decomposition** \( u, v, w \). The demon picks \( u, v, w \) such that \( y = u, v, w \) and \( v \neq \epsilon \).

**Step 4: You pick** \( i \geq 0 \). Construct a string \( xuv^iwz \) that is not in \( A \), for some \( i \geq 0 \). Remember that you may want to set \( i \) to 0 in order to accomplish this.

Comments

- The pumping lemma states a property of regular languages. You cannot use it to prove a language is regular, but you can use its contrapositive to prove a language is not regular.

As a reminder, here is the pumping lemma in its positive form:

If a language \( A \) is regular, then there exists a \( k \geq 1 \) such that for all strings \( x, y, z \) with \( xyz \in A \) and \( y \geq k \), there exist strings \( u, v, w \) with \( y = uvw \) and \( v \neq \epsilon \), and for all \( i \geq 0 \), \( xuv^iwz \in A \).

- Be sure your string \( xyz \) is in \( A \) and that \( |y| \geq k \).

- Be sure to handle all possible decompositions of the string \( y \) as \( uvw \). The demon is picking this decomposition, and you cannot pick which decomposition he chooses.

- Don’t choose an \( i \) that is fractional or negative! This is not allowed by the statement of the pumping lemma; \( i \) must be an integer \( \geq 0 \).

- Your string \( xyz \) should somehow depend on the pumping length \( k \). If it doesn’t depend on \( k \), then you cannot guarantee that it will be long enough for all possible values the demon provides.
Example

Consider the language \( A = \{ a^n b^n \mid n \geq 0 \} \). We claim that \( A \) is not regular.

Proof. We will show that \( A \) is not regular using the contrapositive of the pumping lemma. That is, we will show that the pumping lemma properties do not hold, and therefore \( A \) is not regular.

1. The demon chooses some pumping length \( k \geq 1 \).
2. We select \( x = \epsilon, y = a^k \), and \( z = b^k \). Then \( xyz = a^k b^k \in A \), and \( |y| \geq k \).
3. The demon now picks \( u, v, w \) such that \( y = uvw \) and \( v \neq \epsilon \). Without loss of generality, suppose the demon picks \( u, v, w \) of lengths \( j, m, n \), respectively. Then \( k = j + m + n \) and \( m > 0 \), and \( y = a^j a^m a^n \).
4. But whatever the demon picks, we can win by taking \( i = 2 \):
   \[
   xuv^2wz = a^j a^2m a^nb^k = a^{j+2m+n}b^k = a^k a^m b^k,
   \]
   which is not in \( A \) because there are different numbers of \( a \)'s and \( b \)'s.