

$$\begin{aligned}
f_{h,\varepsilon}(x,y) &= \varepsilon \mathbf{E}_{x,y} \int_0^{t_\varepsilon} L_{x,y_\varepsilon(\varepsilon u)} \varphi(x) \, du \\
&= h \int L_{x,z} \varphi(x) \rho_x(dz) \\
&\quad + h \left[ \frac{1}{t_\varepsilon} \left( \mathbf{E}_y \int_0^{t_\varepsilon} L_{x,y^x(s)} \varphi(x) \, ds - t_\varepsilon \int L_{x,z} \varphi(x) \rho_x(dz) \right) \right. \\
&\quad \left. + \frac{1}{t_\varepsilon} \left( \mathbf{E}_y \int_0^{t_\varepsilon} L_{x,y^x(s)} \varphi(x) \, ds - \mathbf{E}_{x,y} \int_0^{t_\varepsilon} L_{x,y_\varepsilon(\varepsilon s)} \varphi(x) \, ds \right) \right] \\
&= h \hat{L}_x \varphi(x) + h \theta_\varepsilon(x,y)
\end{aligned}$$