Introduction to suffix trees

Lecture 3.1

by Marina Barsky

Pattern matching problem - revisited

- KMP is a provable linear-time algorithm for the patter-matching problem
- It works in a situation when the pattern is fixed and the text is streaming – the text is not known before the search starts

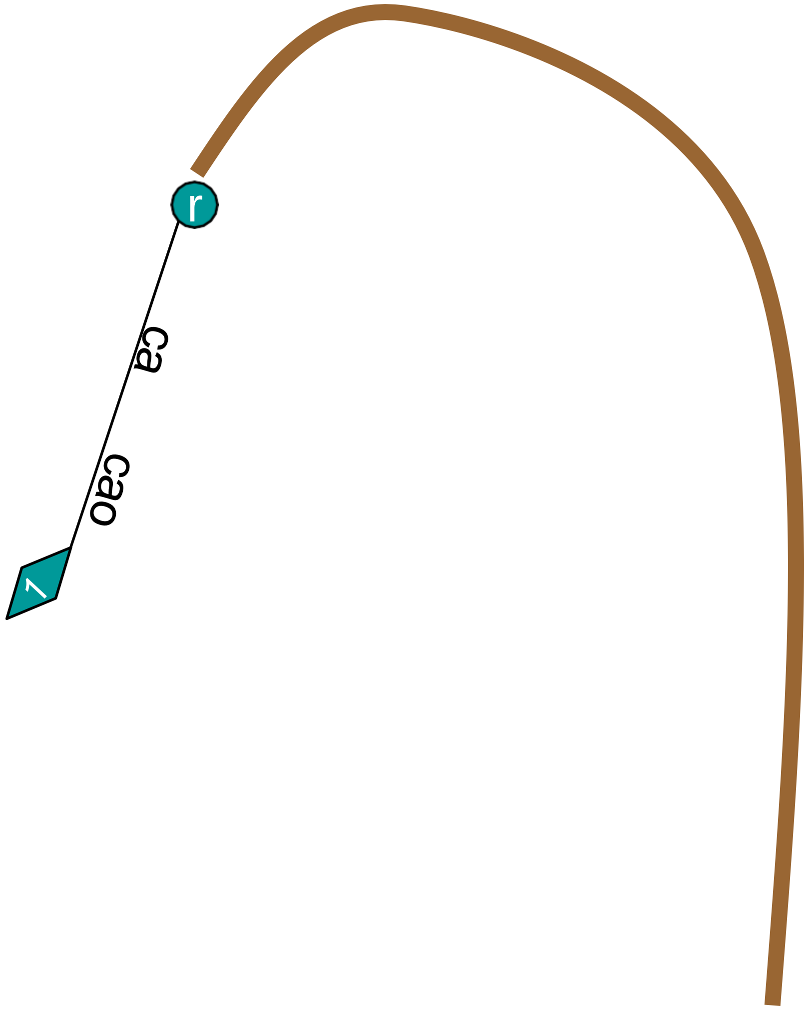
- Let's consider a different scenario:
  - text *T* is known first and it is kept fixed for some time
  - new search patterns are constantly arriving
  - search for each pattern should be as quick as possible

Suffix trees

- Suffix tree of *T* exposes the internal structure of the input text
- Assuming that the text is re-written in a form of the suffix tree, the pattern matching problem can be performed in time $O(M+k)$, where M is the length of a **pattern**, and k is the number of occurrences. The search time *does not depend* on the length of *T*
- In addition, suffix trees provide optimal (linear-time) solutions to numerous complex problems other than pattern matching problem

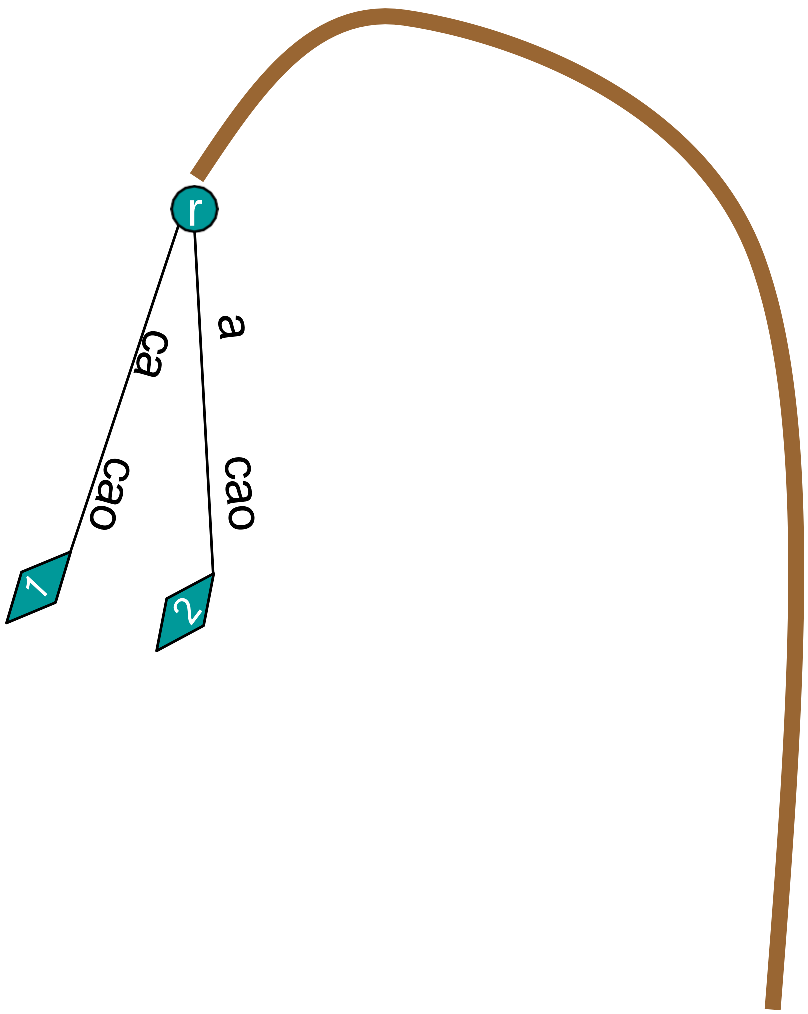
Tree branch with suffixes

T=<u>*cacao*</u>



Tree branch with suffixes

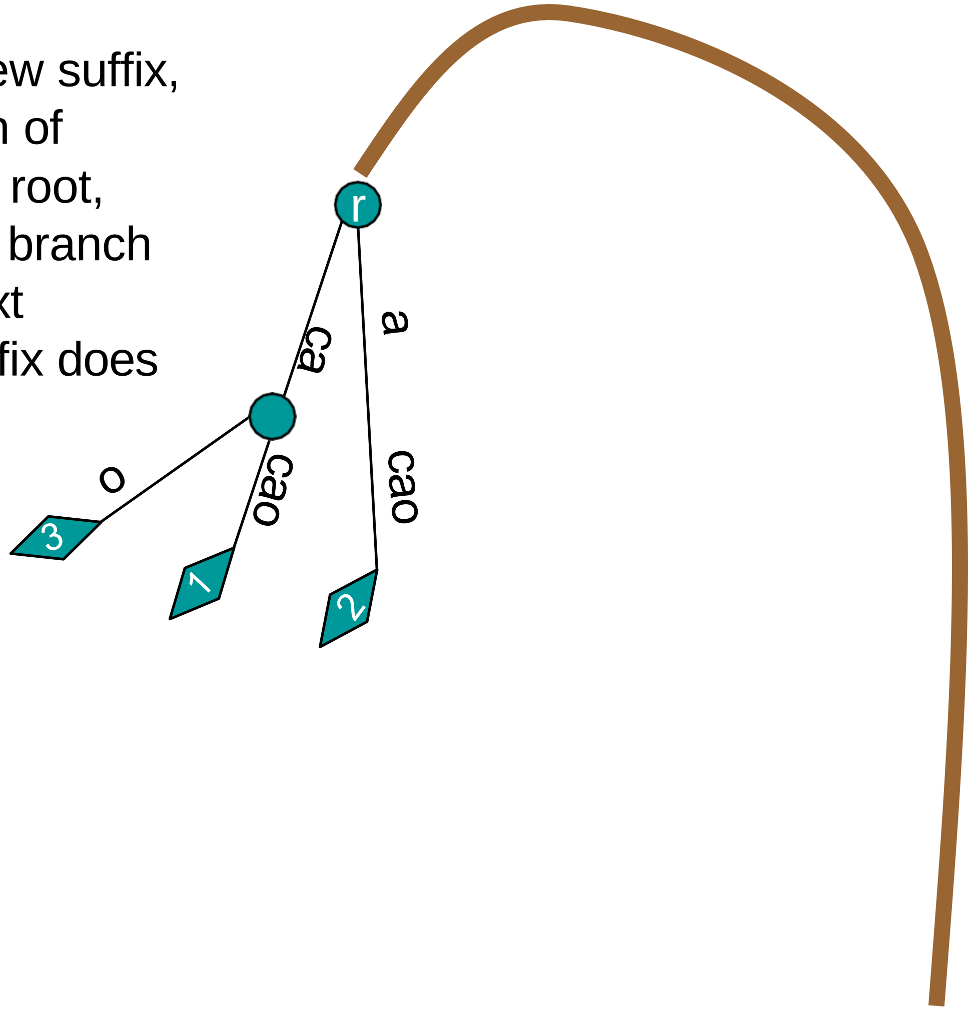
T=*cacao*



Tree branch with suffixes

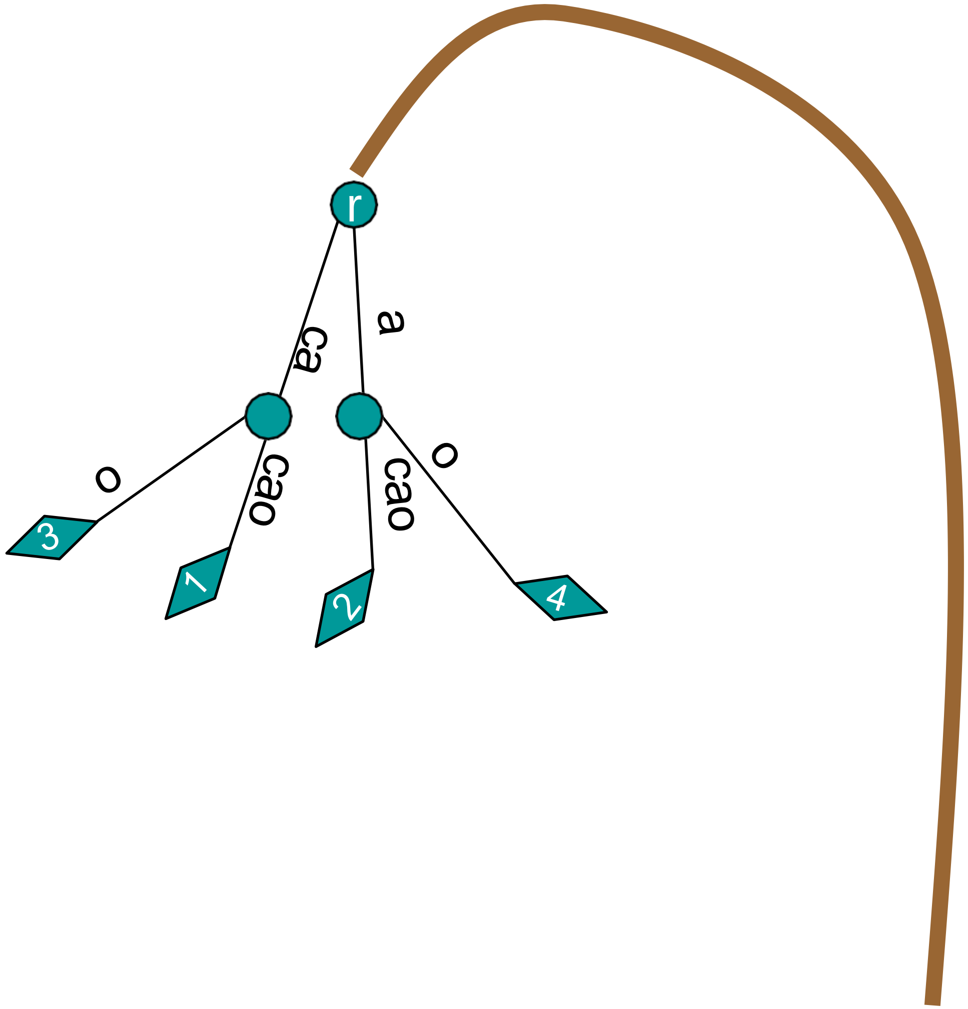
T=*cacao*

While adding a new suffix, we follow the path of matches from the root, and create a new branch only when the next character of a suffix does not match



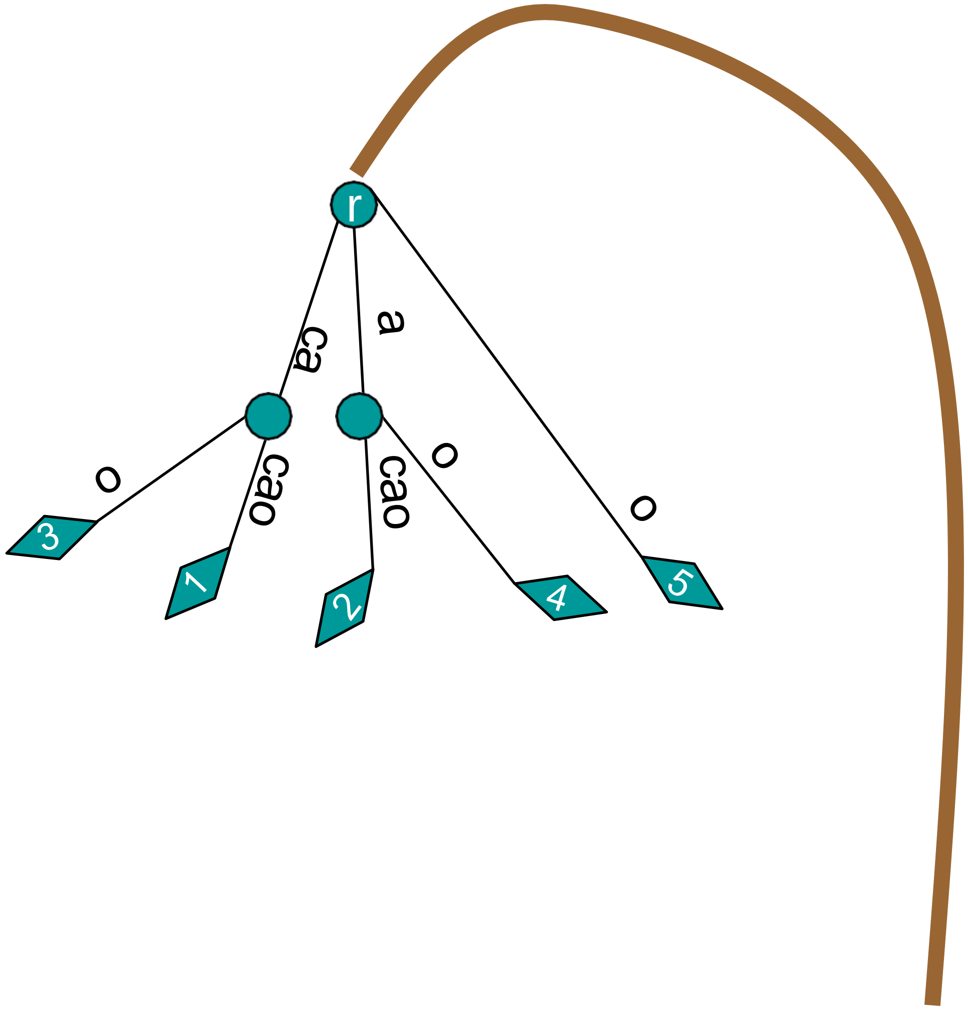
Tree branch with suffixes

T=*cacao*



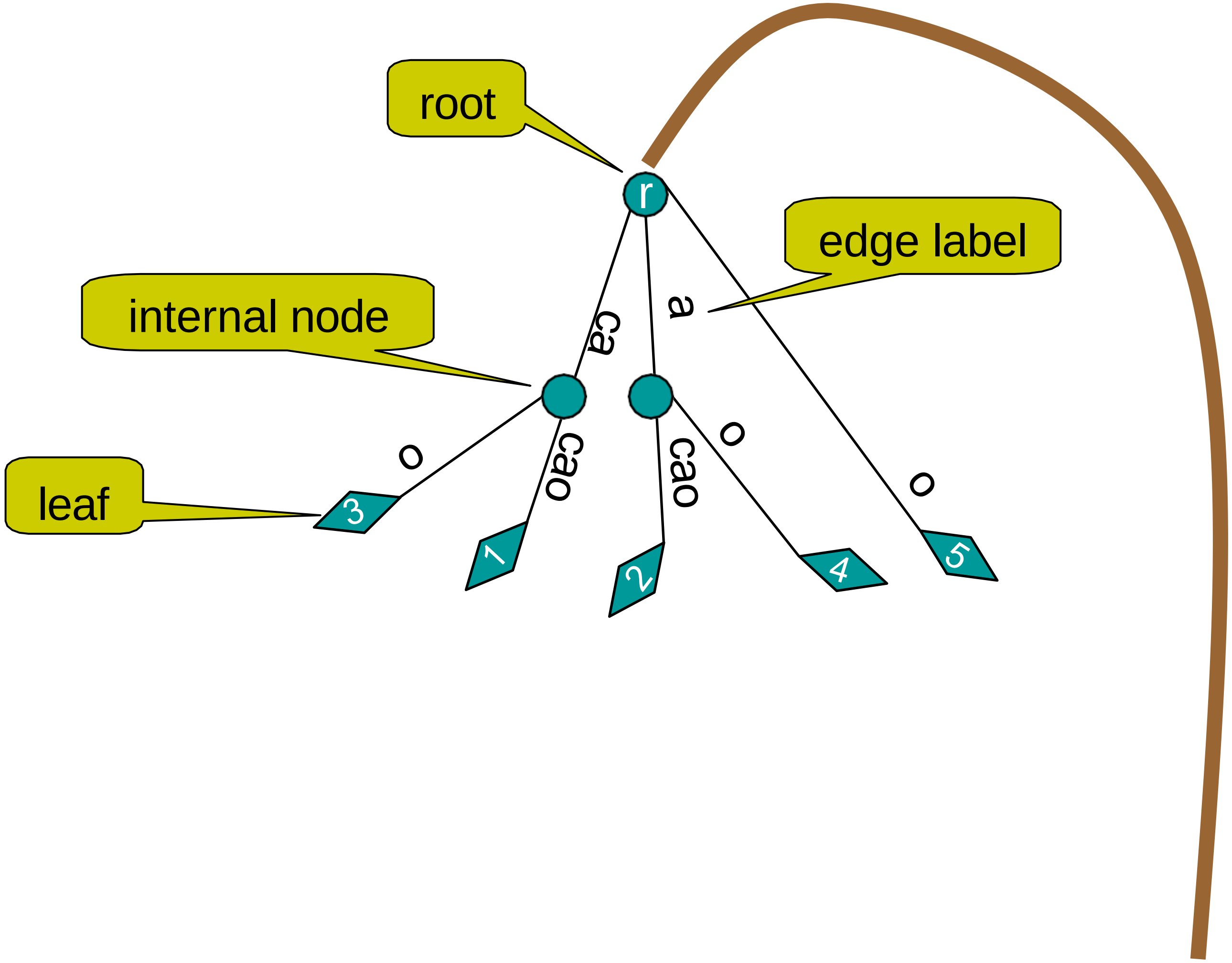
Tree branch with suffixes

T=*cacao*



Suffix tree: terminology

T=*cacao*



Suffix tree - definition

- *A suffix tree* for string *T* (of length *N)* is a rooted tree with the following properties*:*
  - *N* leaves, numbered 1 to *N*.
  - Each internal node has at least two children.
  - No two edges out of a node have edge-labels beginning with the same character.
  - For any leaf *i*, the concatenation of the edge- labels on the path from the root to leaf *i* spells out the suffix *T*[*i*..*N*] of *T.*

Suffix tree – number of nodes

- *A* *suffix tree* for string *T* (of length *N)* is a rooted tree with the following properties*:*
  - *N* leaves, numbered 1 to *N*.
  - Each internal node has at least two children.

  - Because we go from *N* leaves to 1 root node replacing at least 2 nodes with one, the entire process takes at most log *N* steps: **the height of the suffix tree is at most log *N***
  - ***Corollary***: the total number of nodes in the tree is bounded by $2^{\log N} = O(N)$: *N* leaves and *N* internal nodes

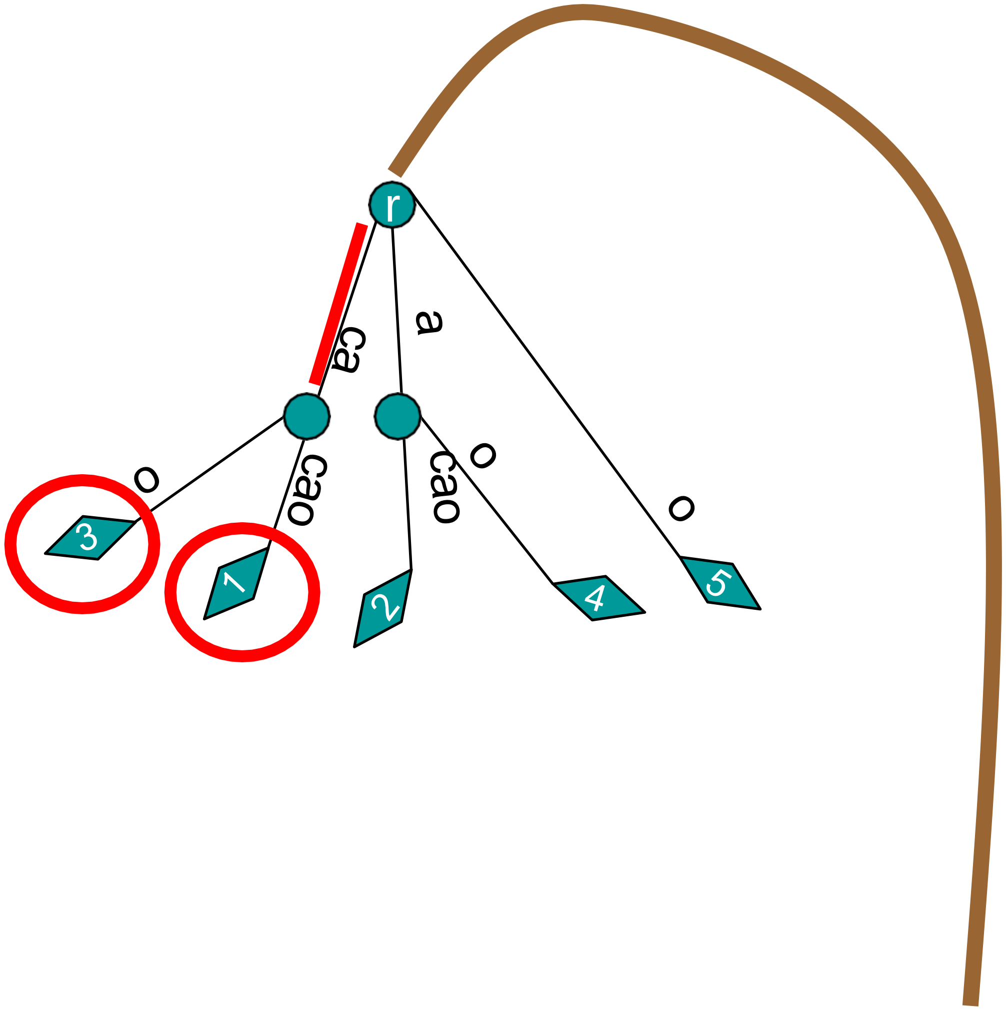
Full-text indexing

- Suffix tree is an example of a ***full-text index*** – the data structure designed for fast search of any substring of a given text

- All different substrings of *T* can be found in the suffix tree following the path from the root

Search for pattern *ca*

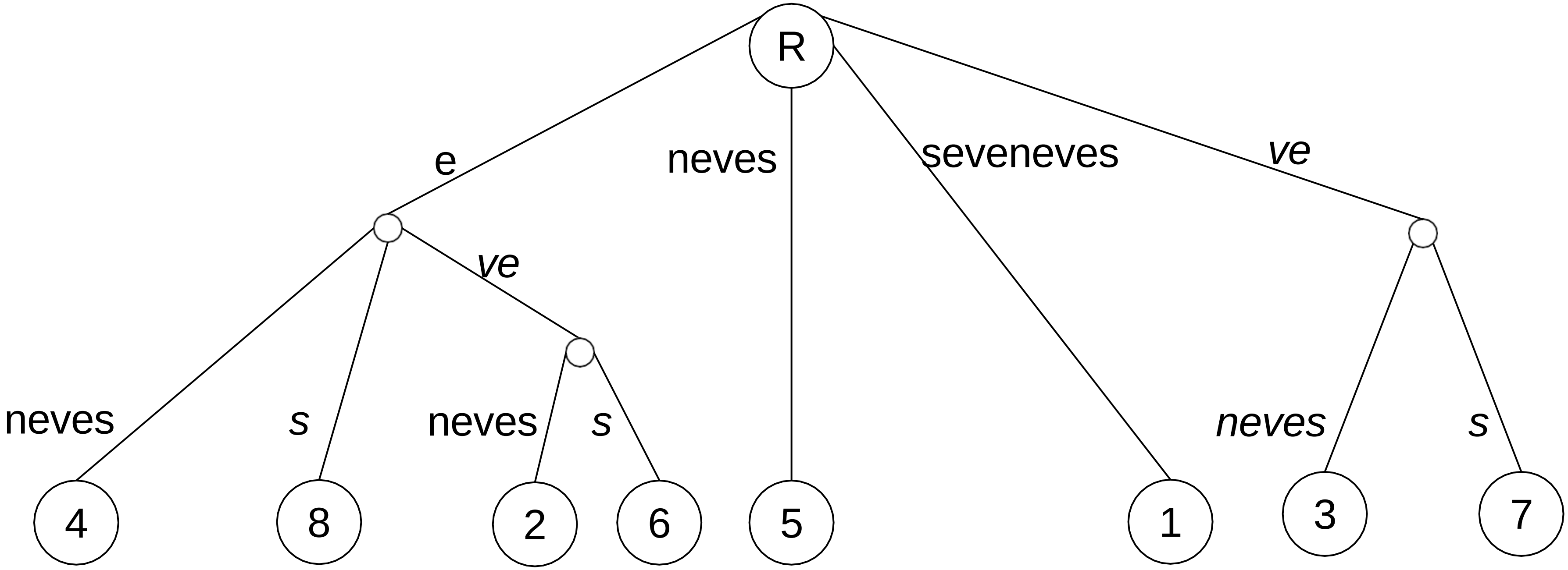
T=*cacao*



Another suffix tree

s e v e n e v e s

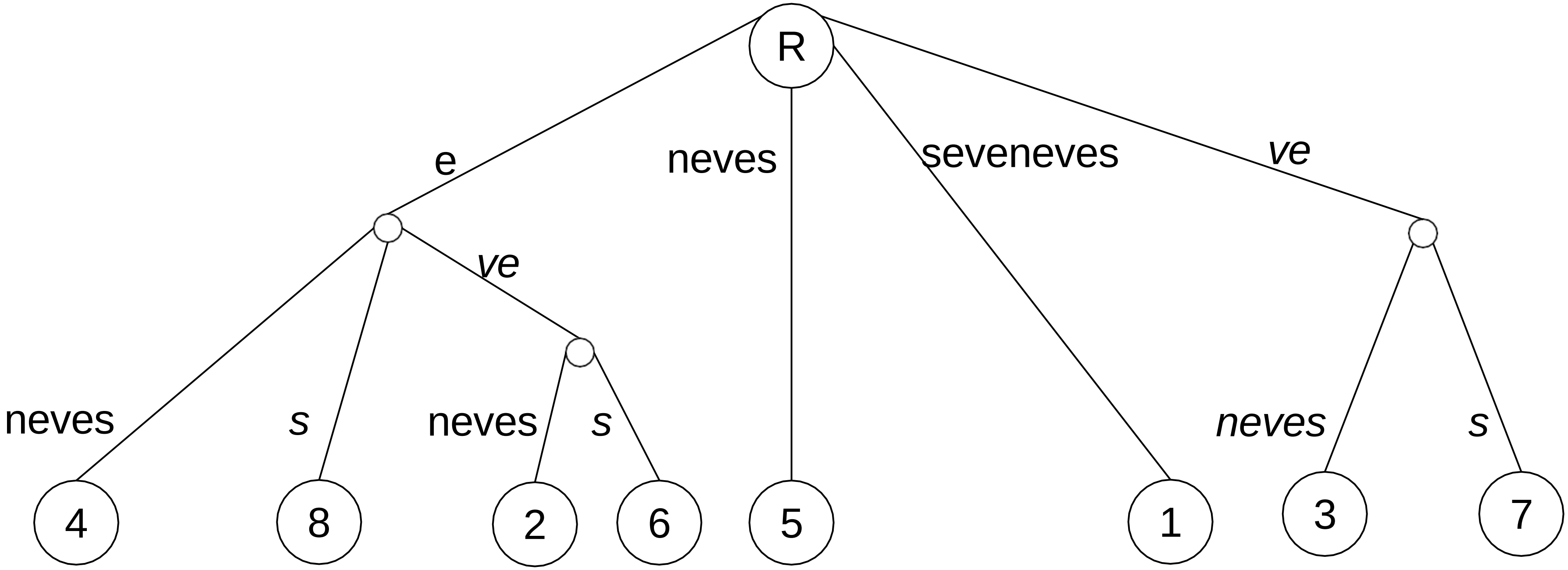
1 2 3 4 5 6 7 8 9



Another suffix tree

s e v e n e v e s

1 2 3 4 5 6 7 8 9

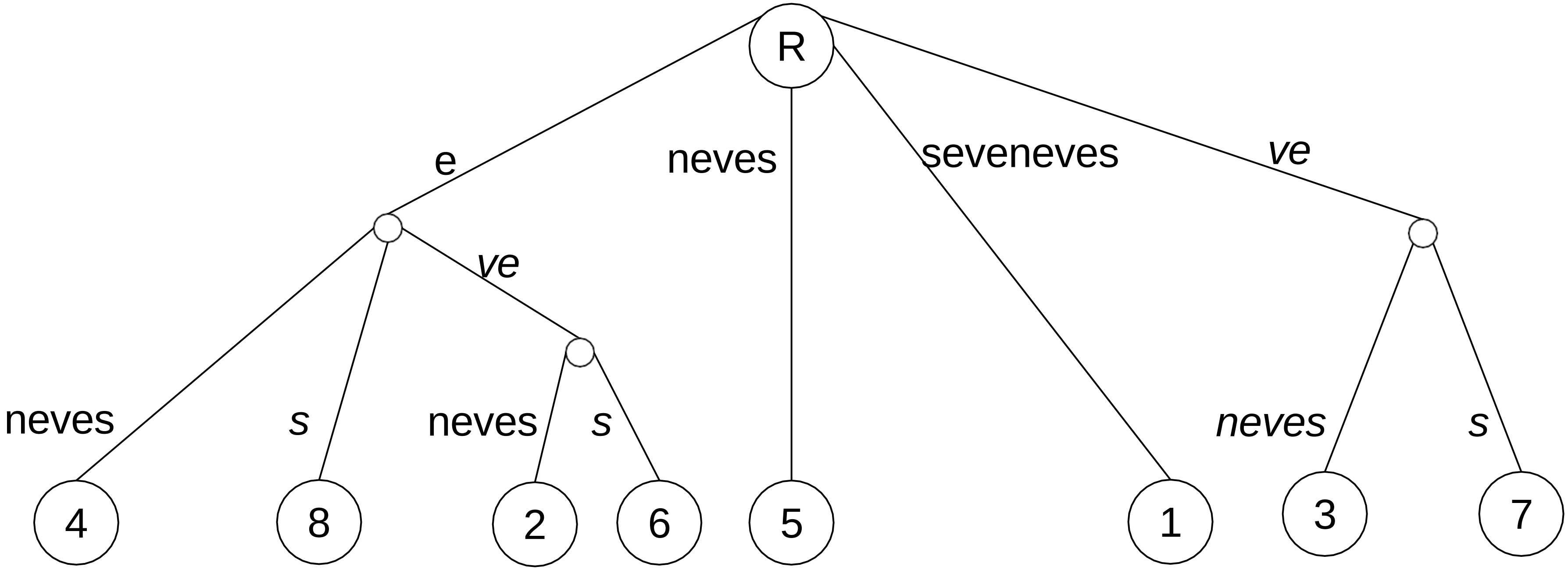


What suffix is missing?

Another suffix tree

s e v e n e v e s

1 2 3 4 5 6 7 8 9



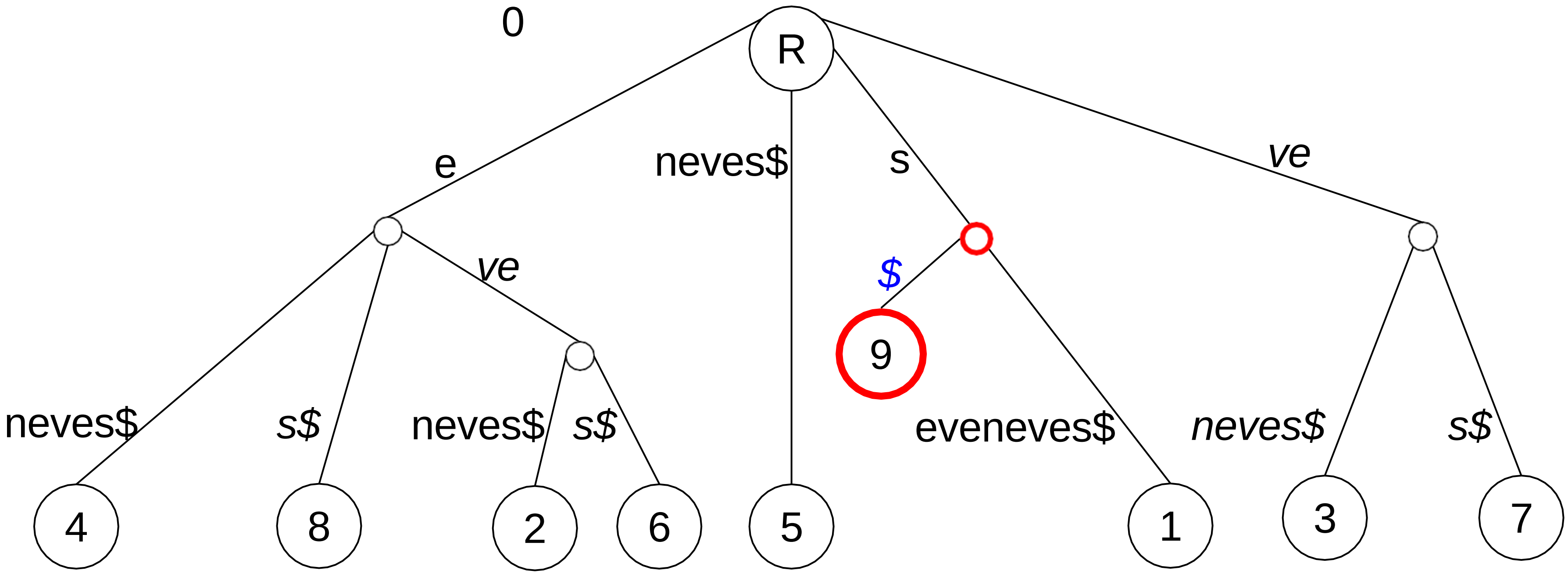
Where is the leaf for $T[9\ldots9]=s$?

What if we search for pattern $P=s$?

Proper suffix tree

s e v e n e v e s $

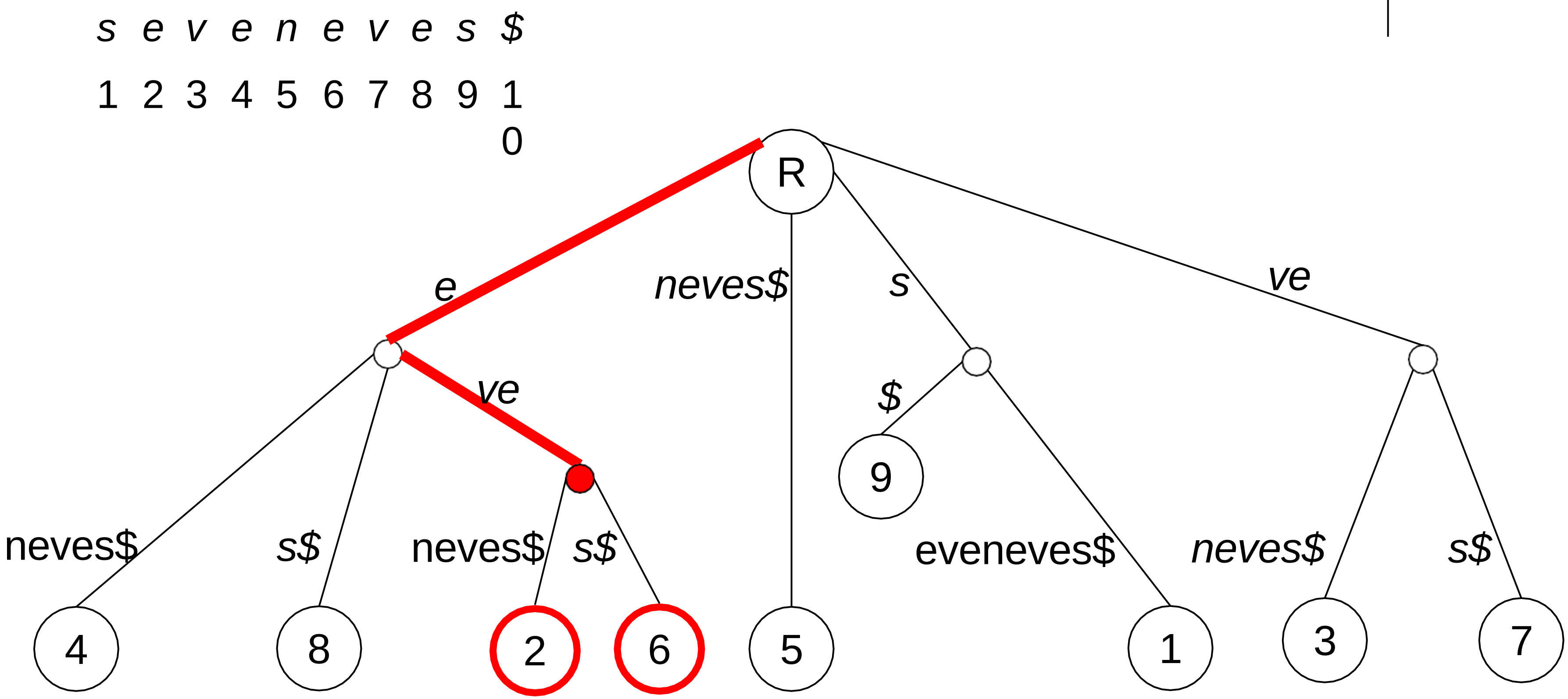
1 2 3 4 5 6 7 8 9 10



We add a special character to the end of T – *sentinel*

The sentinel $ does not occur anywhere in T

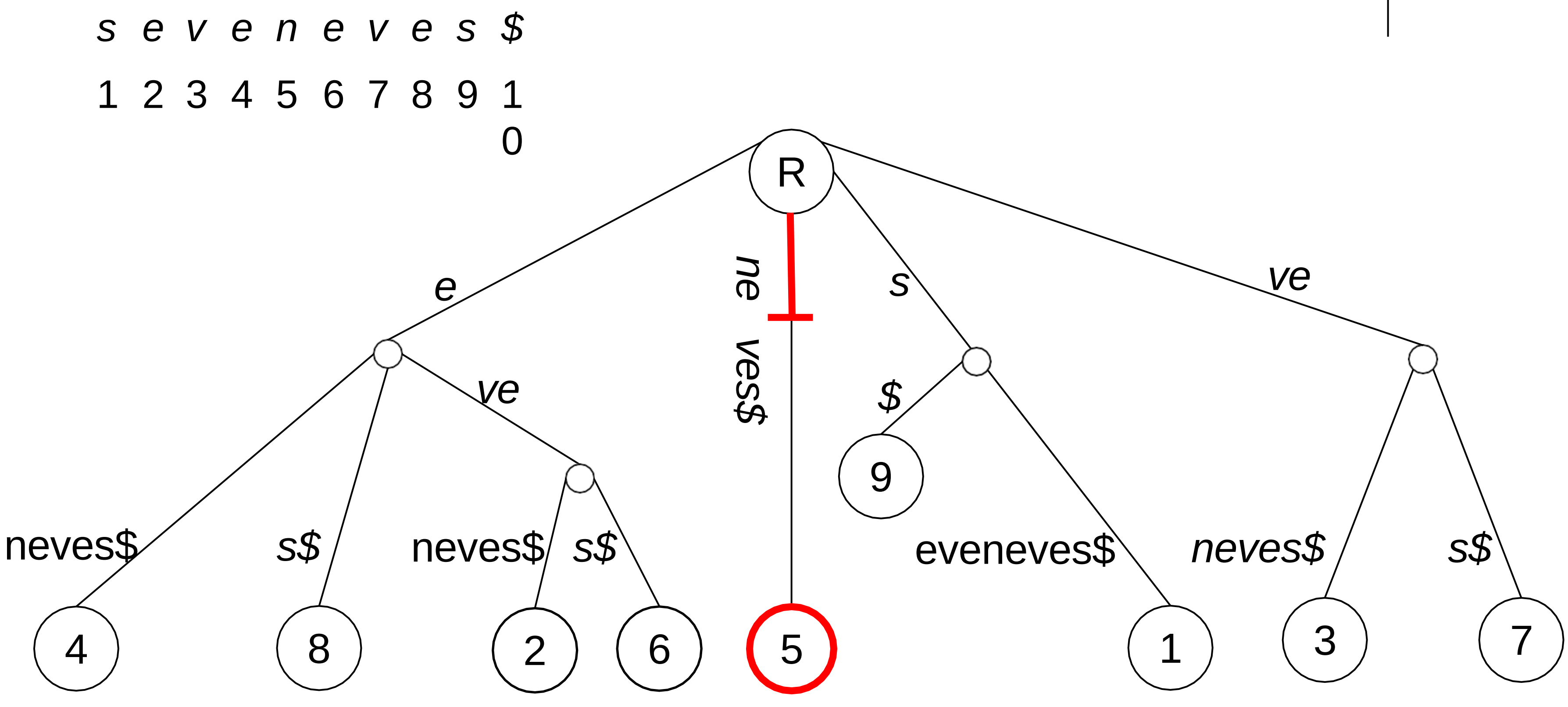
Search for *P=eve*



k – total number of matches

Search in time O(*M*+*k*)

Search for *P*=*ne*



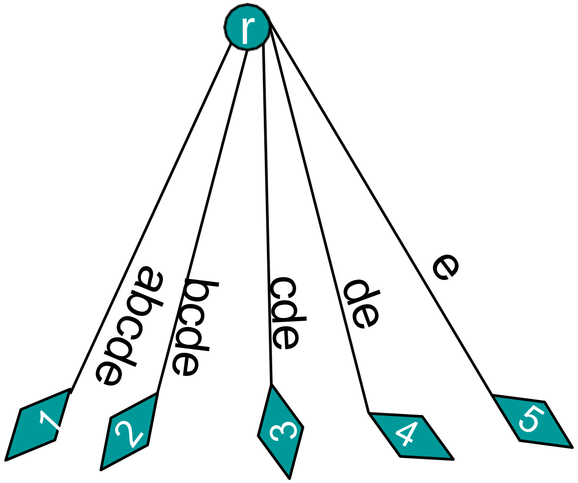
Search in time O(*M*+*k*)

Activity

- build a tree for *T=banana*
- explain how to search for a pattern *ana*

Space

T=*abcde*



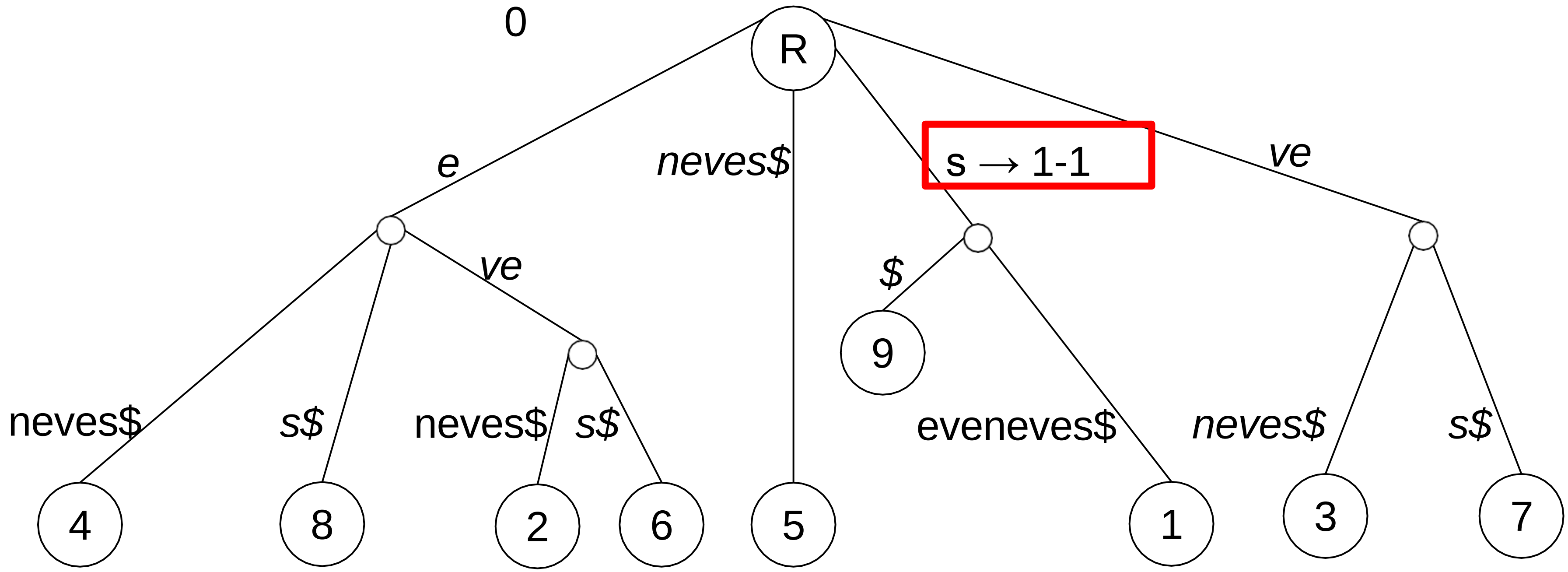
This tree occupies quadratic space!

$1+2+3+\ldots.N=O(N^2)$

Trick – re-label the edges

s e v e n e v e s $

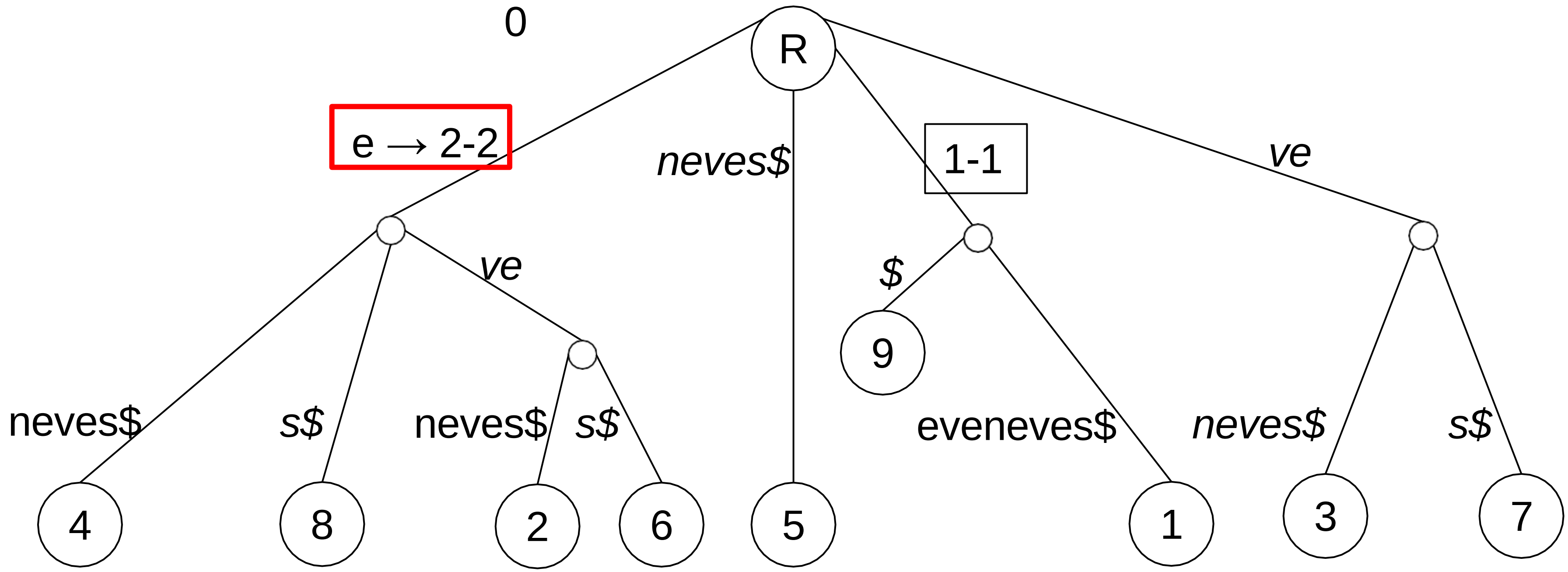
1 2 3 4 5 6 7 8 9 10



Trick – re-label the edges

s e v e n e v e s $

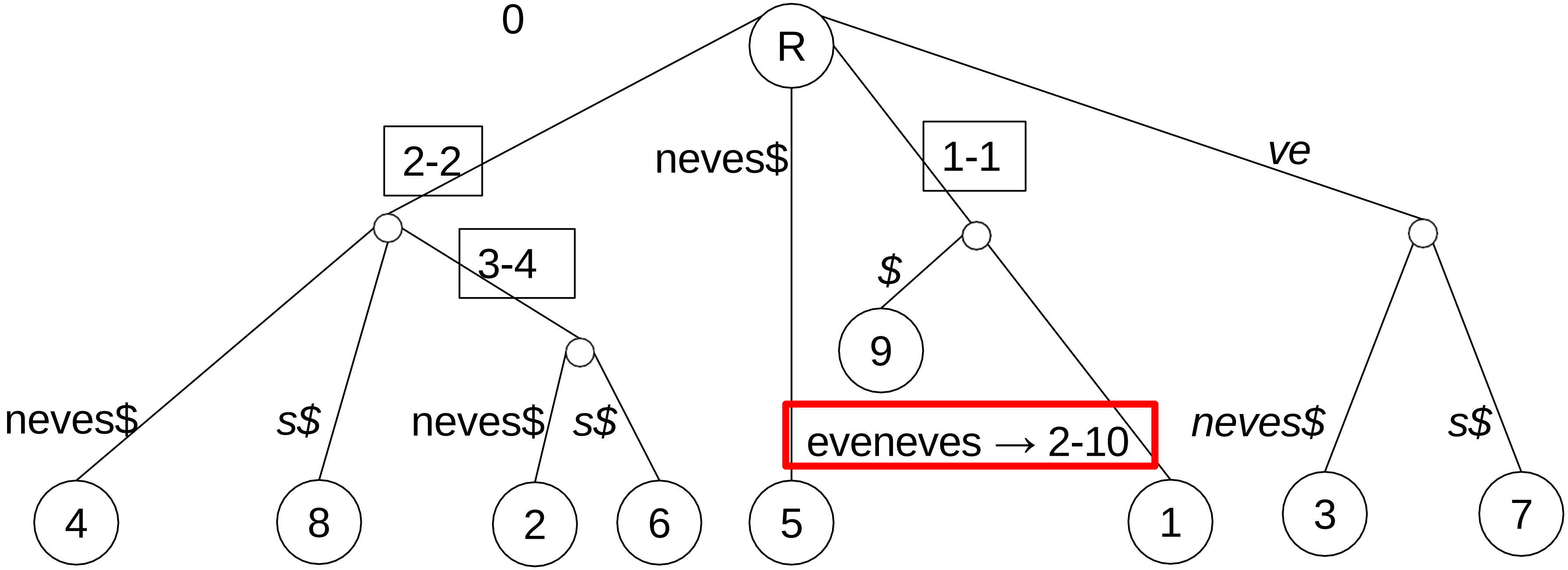
1 2 3 4 5 6 7 8 9 10



Trick – re-label the edges

s e v e n e v e s $

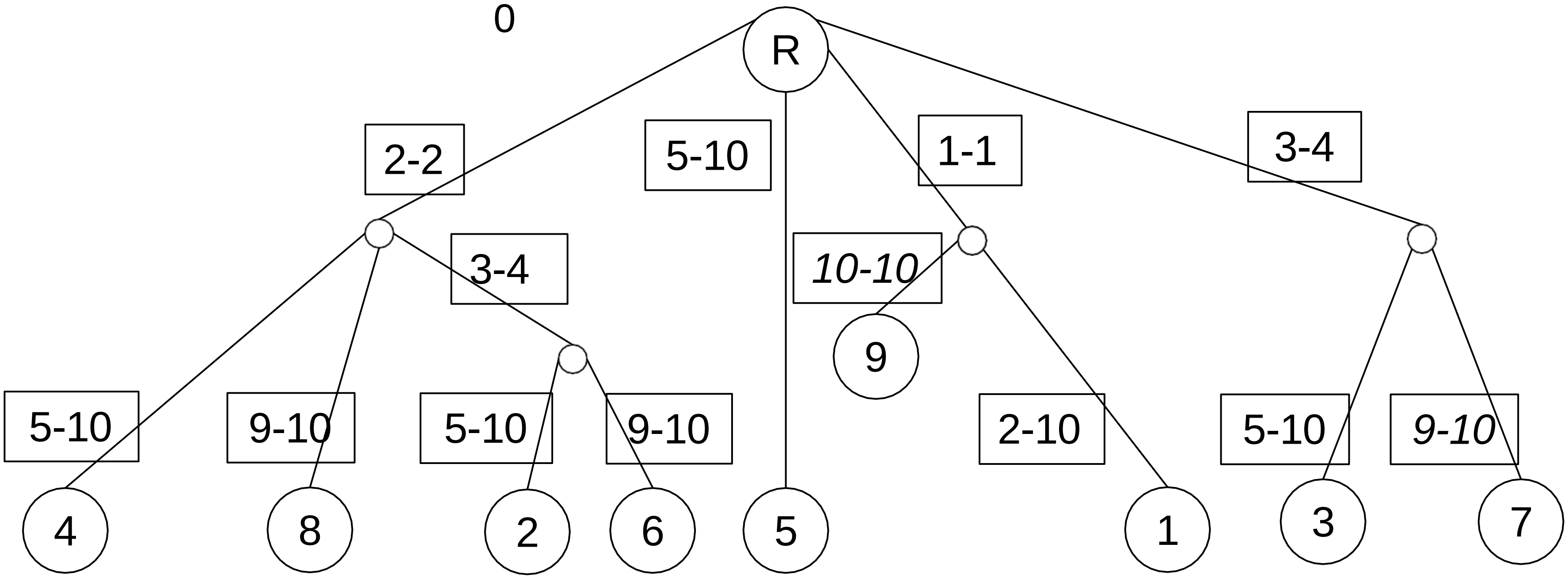
1 2 3 4 5 6 7 8 9 10



Linear space

s e v e n e v e s $

1 2 3 4 5 6 7 8 9 10



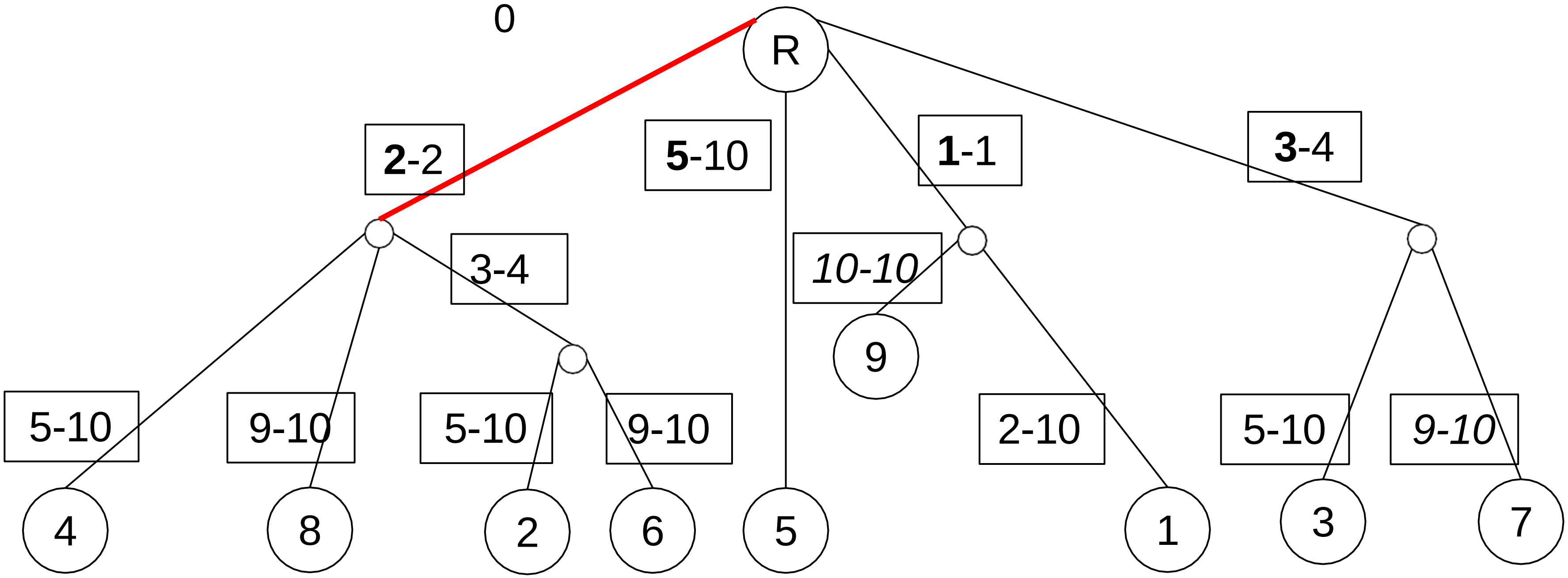
The total number of leaves is O(*N*), the total number of internal nodes is O(*N*)

With a constant storage space per edge – the suffix tree can be stored in linear space

Search

s e v e n e v e s $

1 2 3 4 5 6 7 8 9 10



In order to find an outgoing edge which starts with *e*, we check which of T[2], T[5], T[1] or T[3] is *e*.

The search is as efficient as before, **assuming constant time access to each character of T**

Search with suffix trees: summary

- If we have preprocessed text *T* into its suffix tree, we can answer a Boolean query about an occurrence of a pattern of length *M* by performing only *M* steps, <u>independently</u> of the length of the text *T*
- In order to report all *k* occurrences of a pattern, the traversal of a corresponding subtree is performed in O(*k*) steps

Readings

- Text book Chapter 5

- http://en.wikipedia.org/wiki/Suffix_tree
- http://www.allisons.org/ll/AlgDS/Tree/Suffix/