# Applications of Suffix Trees: Longest common substrings 

Lecture 3.3
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## The longest common substring of several strings

- The problem: find the longest substring common to two given strings I and II.
- For example, if I=superiorcalifornialives and II=sealiver, then the longest common substring of I and II is alive.
- 1970 - Knuth conjectured that the linear-time solution to the longest common substring problem would be impossible


## The longest common substring for 2 strings in linear time

- Concatenate 2 strings and build the suffix tree for the concatenated string
- Label each leaf with the corresponding suffix start position, plus the ID of the string (I or II)
- Perform the depth-first traversal and mark each internal node by I, II or both, depending what suffixes are found in the subtree for this node
- Find the deepest internal node which is marked by both I and II


## Example: I=bananas II=canal



| $b$ | $a$ | $n$ | $a$ | $n$ | $a$ | $s$ | $c$ | $a$ | $n$ | $a$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 1 | 1 |

## Example: I=bananas II=canal



| $b$ | $a$ | $n$ | $a$ | $n$ | $a$ | $s$ | $c$ | $a$ | $n$ | $a$ | $l$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 1 | 1 | 1 |
|  |  |  |  |  |  |  |  |  | 0 | 1 | 2 |



## Example: Marking internal nodes



| $b$ | $a$ | $n$ | $a$ | $n$ | $a$ | $s$ | $c$ | $a$ | $n$ | $a$ | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 1 | 1 | 1 |
|  |  |  |  |  |  |  |  | 0 | 1 | 2 |  |



## Example: What is the longest common substring?



| $b$ | $a$ | $n$ | $a$ | $n$ | $a$ | $s$ | $c$ | $a$ | $n$ | $a$ | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 1 | 1 | 1 |
|  |  |  |  |  |  |  |  |  | 0 | 1 | 2 |



## Example: LCS=ana



| $b$ | $a$ | $n$ | $a$ | $n$ | $a$ | $s$ | $c$ | $a$ | $n$ | $a$ | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 1 | 1 | 1 |
|  |  |  |  |  |  |  |  |  | 0 | 1 | 2 |



# Longest common substrings: example 



Query: what do tiger and pigeon have in common?

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# Common substrings for a set of DNA sequences 

Insert suffixes of multiple strings into one tree

- Discover substrings common to viruses and humans
- Discover substrings unique to cancer

Used in the identification of the remains of US military personnel

- Mitochondrial DNA from live person is collected, sequenced and the sequences are stored in the database (I)
- Later, the DNA is extracted from the remains (II), and the longest common substring of I and II helps to narrow down the search



## Time series as strings

SAX - Symbolic Aggregate approximation (by Eamon Keough, 2001)


## Suffix trees for time series



## Suffix trees for time series



Query: what happened after op?

# Suffix trees for time series: rise and fall of stocks 

$50 \%$ po, $50 \%$ spo


Query: what happened after op?


## Inverted index

Query: What animal "eats mouse"

WORD-BASED INDEX

(inverted index):


Answer is in documents 1,2,3

## Meaningful search: example

Collection of 1-sentence documents

- mouse eats cheese (1)
- cat eats mouse (2)
- snake eats mouse (3)



## Meaningful search: example

Query: What animal "eats mouse"
Collection of 1-sentence documents

- mouse eats cheese (1)
- cat eats mouse (2)
- snake eats mouse (3)

The answer is in documents 2 and 3 , but not in 1


## Suffix tree for melodies ...



Saint-Saëns, Camille (1835-1921), Carnaval des Animaux, Orch. \& 2 Pfts., Aquarium


Beethoven, Ludwig Van (1770-1827), Für Elise, Pft.

ED\#ED\#EAED\#EFDC\#DECHCDH (S-S ED\#ED\#EHDACEAHEG\#C (B)

## Suffix tree for melodies and plagiarism detection



## Indexing melodies...

## Song 1

```
F [BALLADE]
[Zu Strassburg steht ein hohes Haus]
REG[Deutschland / Frankreich, Lothringen]
MEL[-5_ 1_.23_4_ 2_.31
    -5_ 1_.23_4_ 2_231
    3_ 5_5_5_66 5__2_
    2_ 5_4_3_2_ 1_-6_-5
    -5_ 1_2_3_4_ 2__1_//] >>
FCT[Ballade, Braut - Werbung, Erpressung]
```


## Song 2

F[KRIEGS]
[In Boehmen liegt ein staedtchen]
REG[Deutschland, Hessen, Marburg]
MEL[-5 -5_33_3_3
3_ 5_.55_6_ 5_-0
5_ 7_.67_6_6_5
4_3_5_2_5_ 1_0_//] >>
FCT[Staende -, Soldaten -, Kriegs - Lied]

2 folk songs from the Essen Associative Code (EsAC) database http://www.esac-data.org/data/

# ...and plagiarism detection 

 Generalized suffix tree for two songs

## Set your imagination free©

